Theory of Planet Formation

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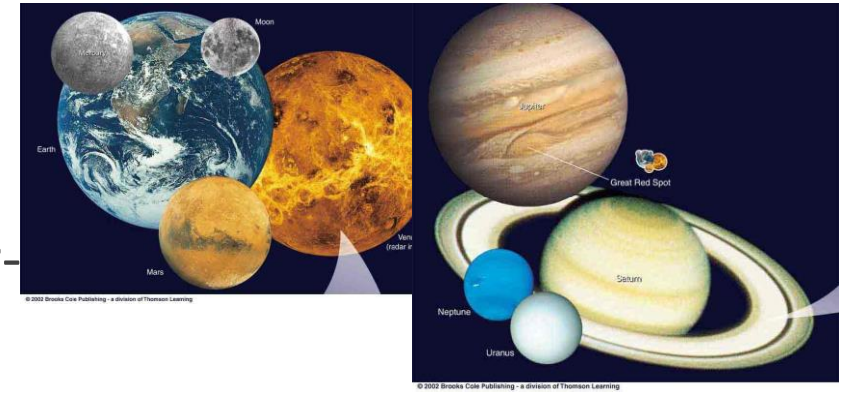
Outline

- What is expected from a planet formation theory
- Early hypothesis
- Nebular Model
- Core Accretion Model
- Gravitational Instability Model
- Migrating Embryo Model
- Tidal forces and tidal downsizing of Migrating Embryos
- Conclusion

Solar system theory; What is expected?

- It should be able to explain the formation of two kinds of planets!
  1) Terrestrial 2) Gas Planets
- It should be able to explain the almost planar structure of solar-system



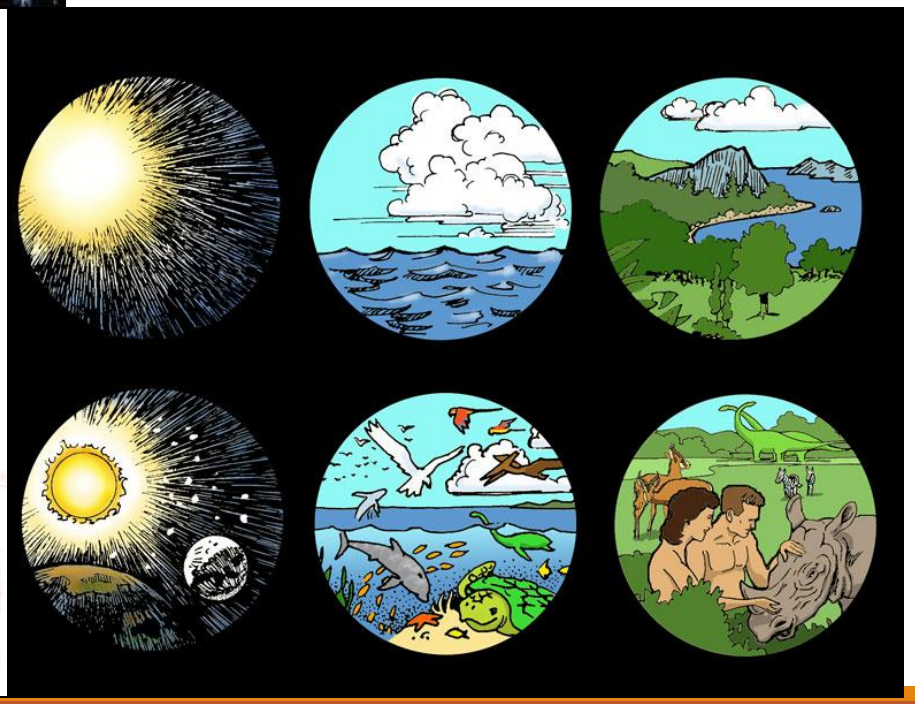
- Domination of solar mass over other masses in system
- All planets revolving in the same direction!

THE SAME LOGIC CAN GIVE MORE THEORIES !!

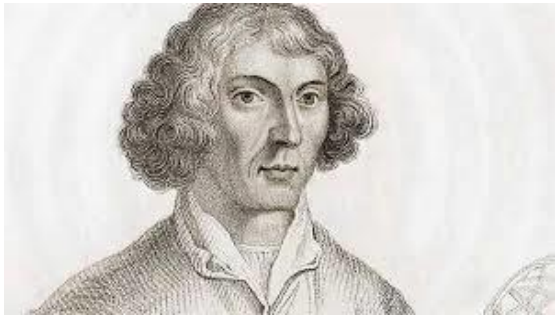
thesis

!!!!!

https://www.youtube.com/watch?v=4D3EREfKnoA



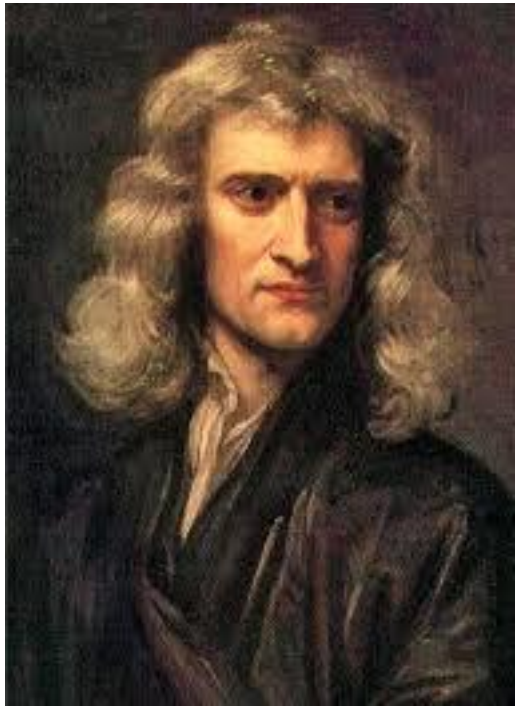
The Enlightenment Age!



Nicolaus Copernicus
(1473-1543)
Solar Centric Universe

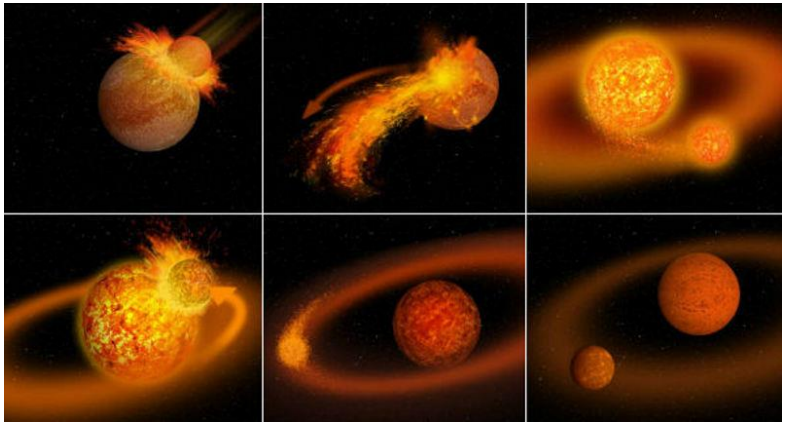
Johannes Kepler
1571-1630

Galileo Galilei
1564-1642



Sir Isaac Newton
1643- 1727

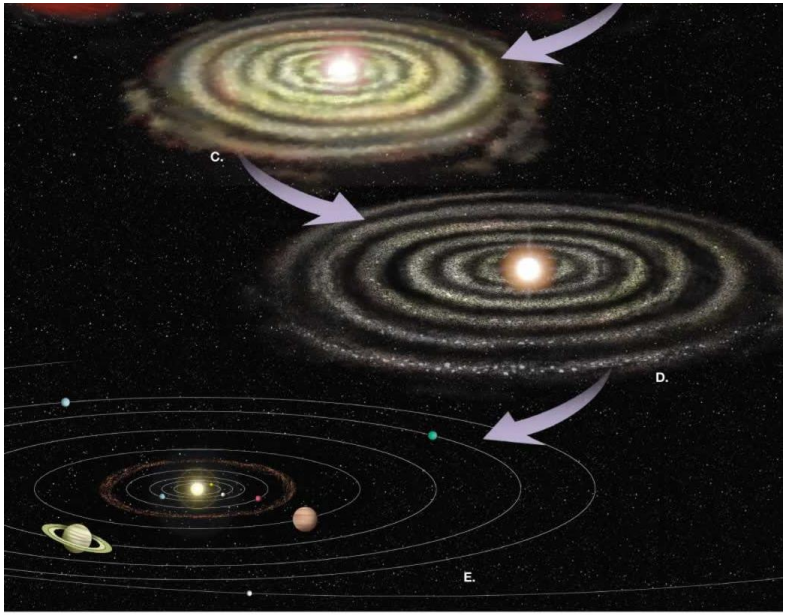
Early Hypothesis



Pic credit : Google

- Catastrophic Hypothesis
  Passing star hypothesis (Georges-Louis de Buffon 1745):
- A star passed near to sun tidally tore materials out of sun and created planets
- Problem: Only few stars would have planets

Early Hypothesis



Pic credit : Google

- Evolutionary Hypothesis
  nebular hypothesis (Pierre-Simon de Laplace 1796):
- Rings of material separate from the spinning cloud, carrying away angular momentum of the cloud → cloud could contract further (forming the sun)
- Problem: Almost all of stars should have planets

Laplace's Nebular Hypothesis

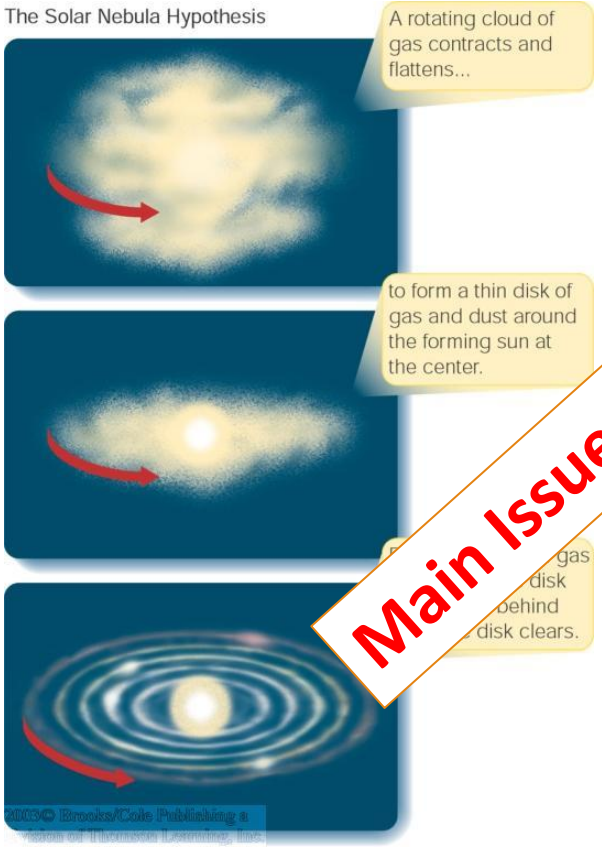
Pierre-Simson Laplace (1749-1827)

First Scientist to talk about Scientific Determinism

"We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes."

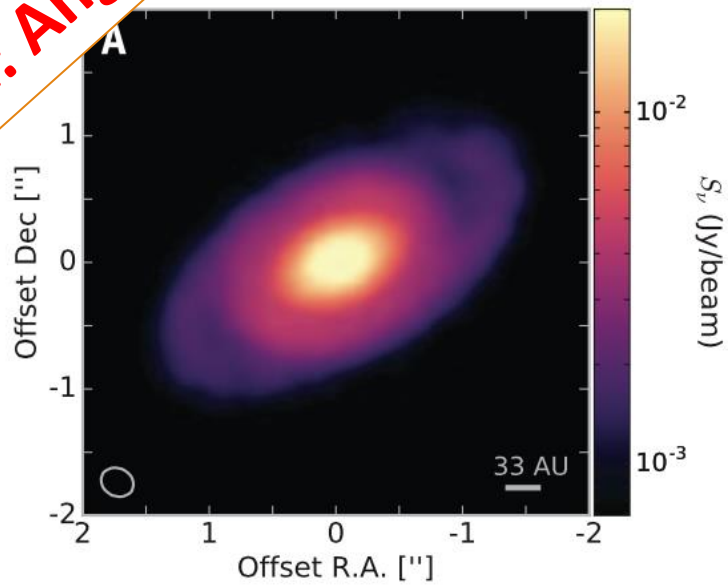
— Pierre Simon Laplace, A Philosophical Essay on Probabilities

Laplace's Nebular Hypothesis



- It's th… of all modern planet fo… theories
- …planets are formed almost at …e same time in the disk
- …e formed

Main Issue: Angular Momentum



Disk around Elias-2-27

Perez et.al 2016

Laplace's Nebular Hypothesis

- Specific Angular Momentum of a rotating cloud is given by

$$j(r) = \Omega(r) r^2$$

Ω(r) – Angular velocity of the cloud at r.

- Centrifugal barrier is given by,

$$r_c = j^2 / GM_*$$

G – Gravitational Constant
M_* - Mass of Sun

- Any plausible cloud has most mass and j(r) in outermost radii.
- But solar system has 99% of mass in the center (Sun) and more than 99% of angular momentum in outer parts (planetary orbits).

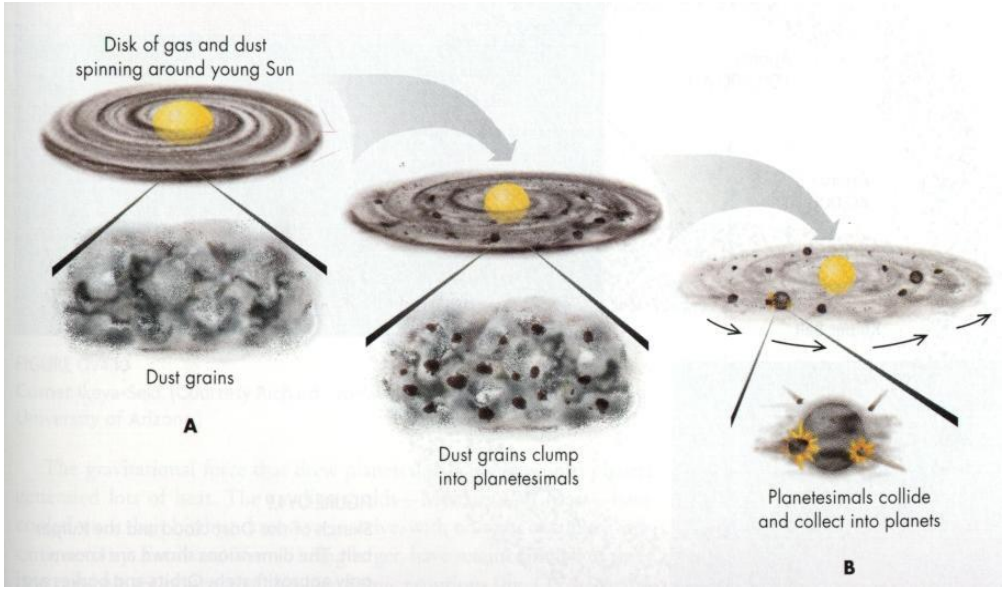
Mostly Abandoned !!

But the idea that planets are formed in the disk remained !

The Core- Accretion Model

Proposed by Safronov (1972)

Planet forms in different steps

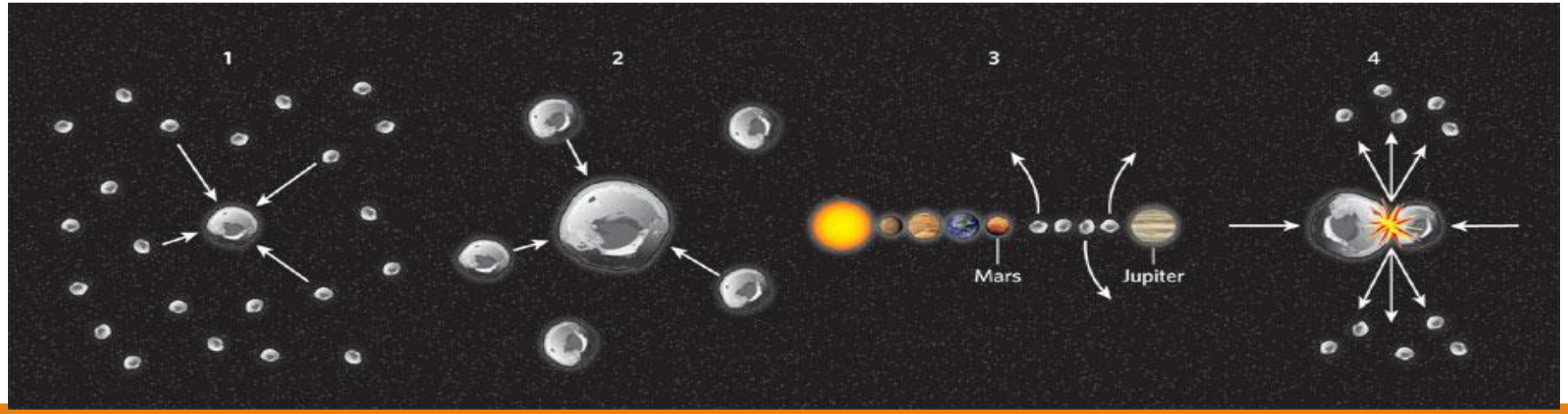


1. As the disk cools, rock and ice grains condense out and settle to the midplane of the disk.
2. Small solid bodies grow from the thin dust layer to form km-sized bodies (***planetesimals***) - gas drag and solar gravity are dominant processes
3. As particle mass increases, their gravity becomes important. Planetesimals collide and grow to become planets or planetary cores – gravitational scattering and solar gravity are dominant processes.

Credit: Dr. Basu's Slides

The Core- Accretion Model

4. A few planetesimals grow large enough to dominate evolution: *planetary embryos*
5. On much slower timescales, planetary embryos collide and grow into *planetary cores* or *proto-planets*
6. Cores of intermediate and giant planets accrete hydrogen/helium gas envelopes



Some Issues (As always!) with the Core Accretion Model

- The time scale needs to form a planet like Jupiter is ~ 10 Million years.
- But most of simulations shows that most of the disk mass will be disappeared by 4-6 million years!
- Also the formation of plenetesimals is still not a completely understood process!
- (Personally, I like models which have their basic assumptions supported by more fundamental theories! ;-))

Gravitational Instability Model

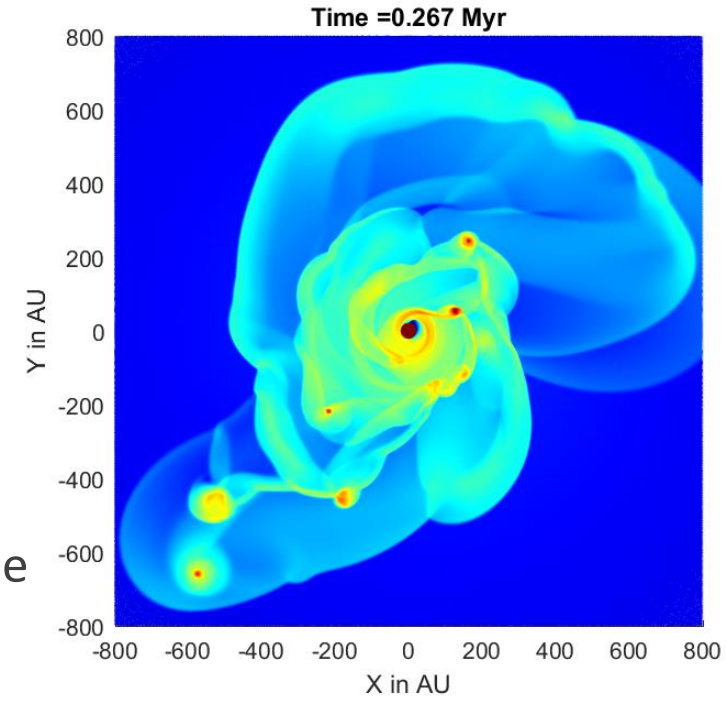
- Gravitational Instability model is really a beautiful model!

Reasons!

1) First because you can start it by writing down basic equations like; Newton's Equation, Mass continuity equation and Energy conservation equation !!

2) The same model can explain the formation of planets, brown dwarfs, stars, galaxies and super clusters!

- In this model planets are formed within the disk due to the gravitational instabilities in disk.



Gravitational Instability

For a molecular cloud under self gravity has to obey the following rules

1) Mass continuity equation: $$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \rho \vec{v} = 0$$

2) Newton's Equation : $$\rho \frac{\partial v}{\partial t} = -\vec{\nabla} P + \rho \vec{g}$$

3) Ideal gas law and polytropic relation: $$P = k\rho^{\gamma}$$

4) Poisson Equation: $$\vec{\nabla} \cdot \vec{g} = -4\pi G\rho$$

$$P = \frac{\rho k T}{m}$$

Gravitational Instability

Now for simple linear analysis assume following;

1) $$\rho = \rho_0 + \rho_1$$
$$g = g_1$$
$$v = v_0 + v_1$$

$$c_s^2 = \gamma \frac{P_0}{\rho_0}$$

Sound Speed

2) We considering only 1 dimension now

$$\frac{\partial^2 \rho_1}{\partial t^2} = c_s^2 \frac{\partial^2 \rho_1}{\partial x^2} + 4\pi G \rho_0 \rho_1$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \rho \vec{v} = 0$$

$$\rho \frac{\partial v}{\partial t} = -\vec{\nabla} P + \rho \vec{g}$$

$$P = k\rho^{\gamma}$$

$$P = \frac{\rho k T}{m}$$

$$\vec{\nabla} \cdot \vec{g} = -4\pi G \rho$$

$$\frac{\partial^2 \rho_1}{\partial t^2} = c_s^2 \frac{\partial^2 \rho_1}{\partial x^2} + 4\pi G \rho_0 \rho_1$$

Gravitational Instability

- To Solve we can use Fourier Analysis!!

We can assume solution is in the form: $\rho_1 = e^{i(\omega t - kx)}$

- Then we get the Dispersion relation :

$$\omega^2 = c_s^2 k^2 - 4\pi G \rho_0$$

Condition for instability : $\omega^2 < 0$

WHY???

Gravitational Instability (Jeans Length)

$$\omega^2 = c_s^2 k^2 - 4\pi G\rho_0$$

$$\omega^2 < 0$$

$$\mathrm{k} < \mathrm{k}_J = \left(\frac{c_s^2}{4\pi G\rho_0}\right)^{-1/2}$$

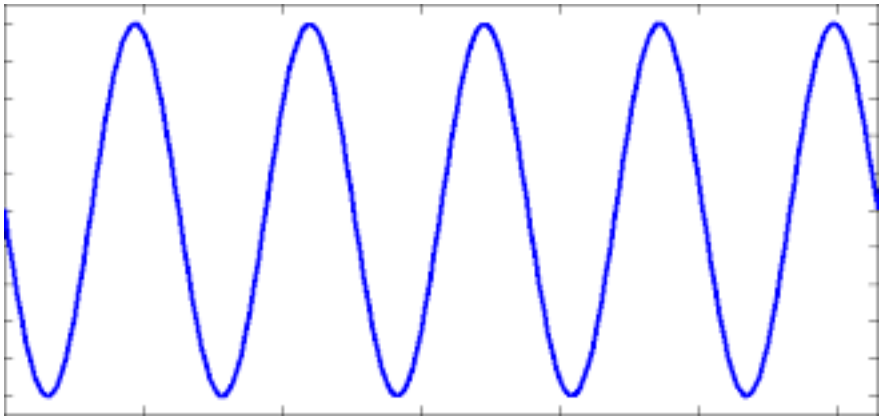
$$k = \frac{2\pi}{\lambda}$$

$$\lambda > \lambda_J = \left(\frac{\pi c_s^2}{G\rho_0}\right)^{1/2}$$

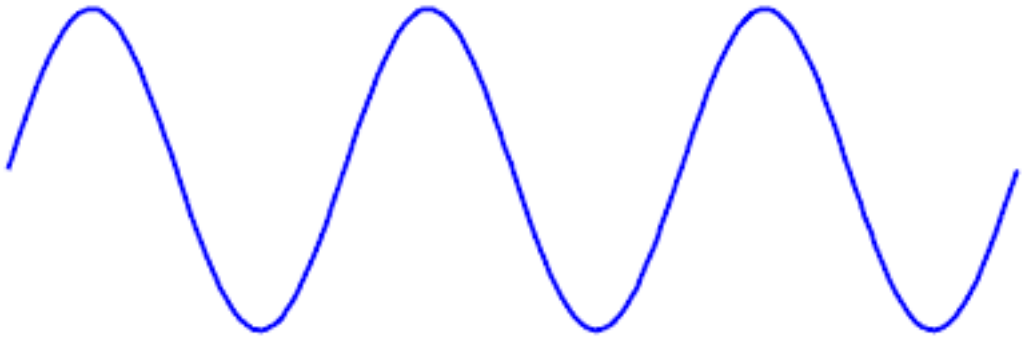
Jeans Length

What does Jeans length means?

$$\lambda > \lambda_J = \left(\frac{\pi c_s^2}{G\rho_0}\right)^{1/2}$$



If perturbation wave length LESS than Jeans length, not enough mass to take over by gravity!!

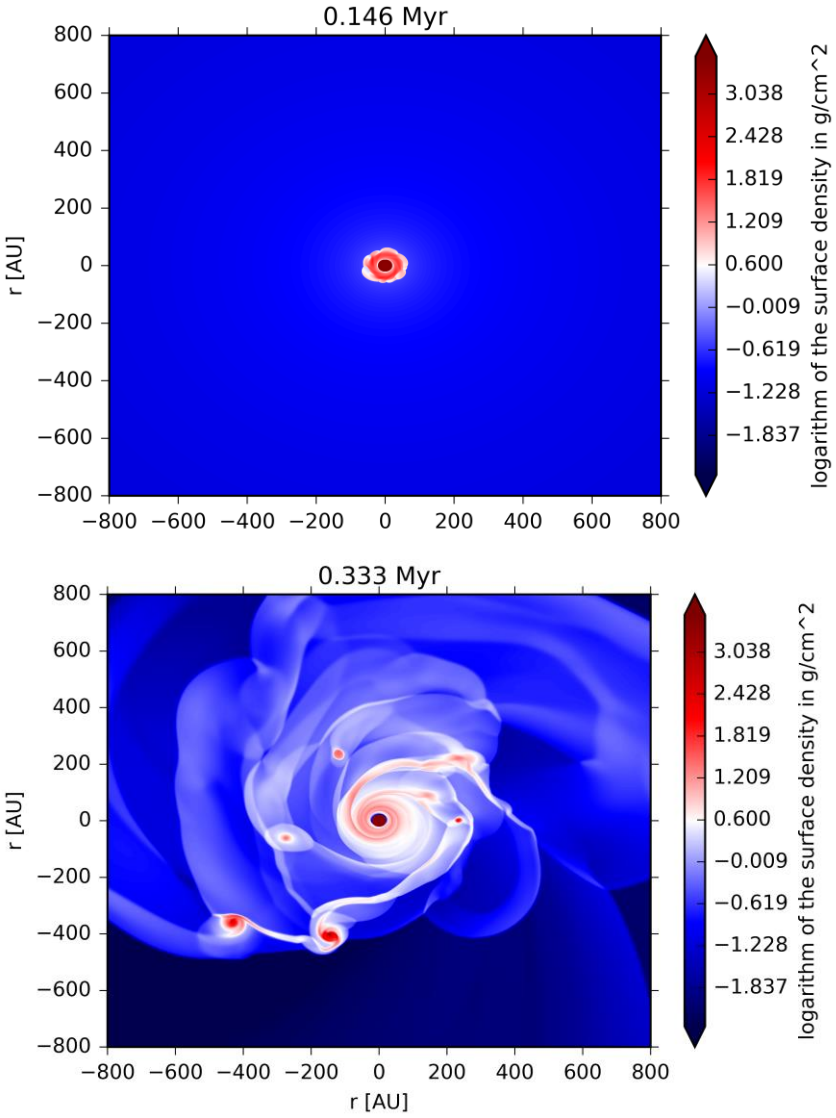


If perturbation wave length MORE than Jeans length, gravity will be dominant over thermal energy and collapses!

Back to Gravitational Instability Model !! No math here! Don't worry ;-)

- During star formation gas cloud collapse to form a protostar and a circumstellar disk around it!
- During the initial few million years disk can get very massive and have disk instabilities and forms fragments or clumps.
- This can easily form planets as massive as Jupiter in 1-2 million year!

Still we have some problems with GI Model!
(Even after doing that whole math!!)



Issues with GI models!!

- Still you can't explain the position of Jupiter and other gas giants in solar system!!
- Jupiter is within 7 AU from sun! In circumstellar disk the mass of clump formed at that distant can't be as big as Jupiter's!
- If r decreases → Temperature in disk increases → Q increases → Stability → No fragmentation !!

$$\lambda > \lambda_J = \left(\frac{\pi c_s^2}{G\rho_0}\right)^{1/2}$$

Toomre' Criteria

$$Q_{\rm gas} \equiv \frac{c_s \kappa}{\pi G \Sigma}.$$

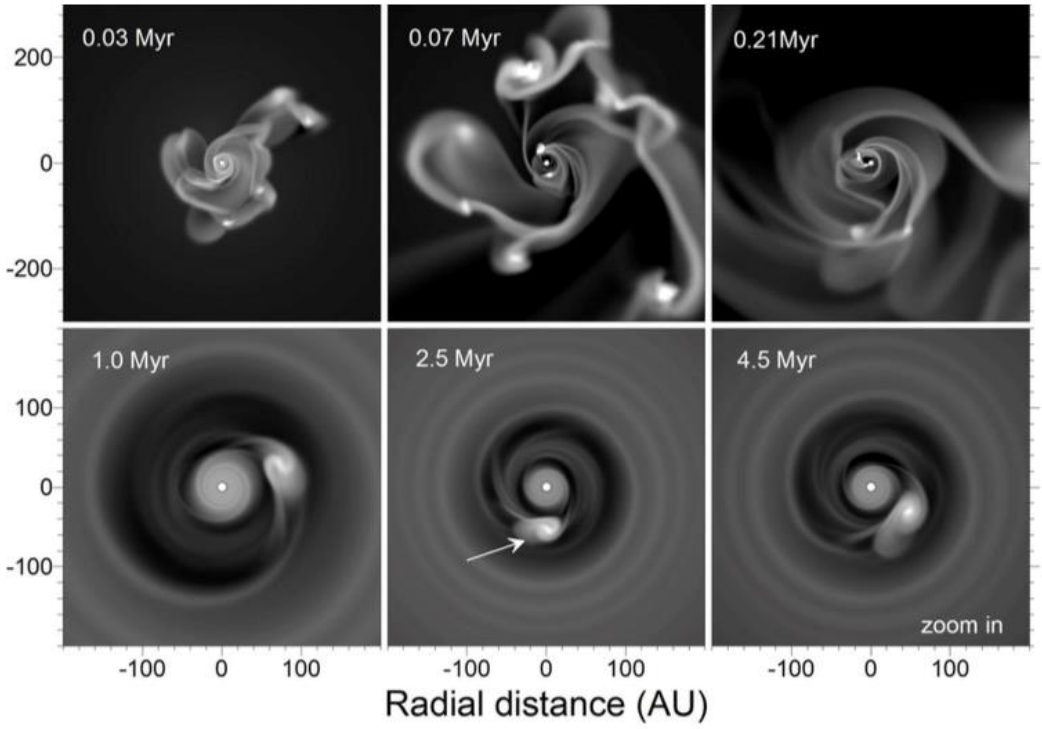
BUT …

$$Q > 1$$ Stability

We have a brand new model !!

Migrating Embryo Model

- It's a disk instability /fragmentation model
- Here massive clumps are formed in wide orbits (R > 100 AU, where Q can be less than 1) with masses more than 13 M_J
- There clumps migrate in to the inner orbits by angular momentum transport !!
- As it comes nearer to the center it can accrete more dust to form solid core
- Finally tidal forces from star will rip away the gas and we get gas giants of mass about ~1M_J



Vorobyov and Basu 2012

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \rho \vec{v} = 0$$

$$\rho \frac{\partial v}{\partial t} = -\vec{\nabla} P + \rho \vec{g}$$

$$P = k\rho^{\gamma}$$

$$P = \frac{\rho k T}{m}$$

$$\vec{\nabla} \cdot \vec{g} = -4\pi G\rho$$

Migrating Embryo Model

- We use a thin disk approximation here,

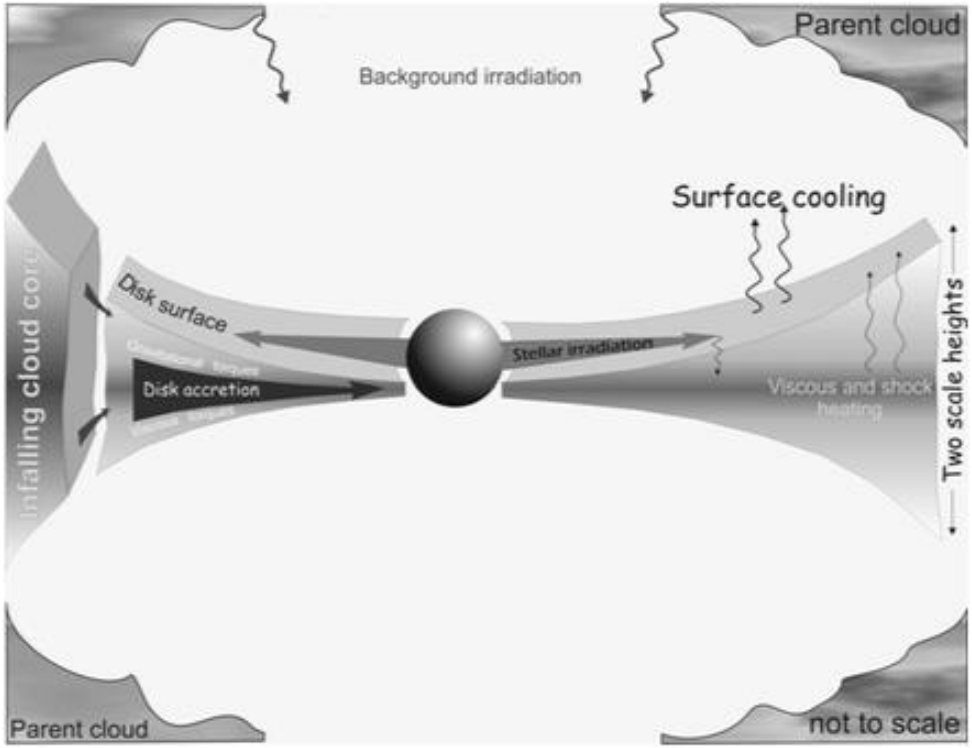
$$\frac{\partial \Sigma}{\partial t} = -\nabla_p \cdot (\Sigma \boldsymbol{v}_p),$$

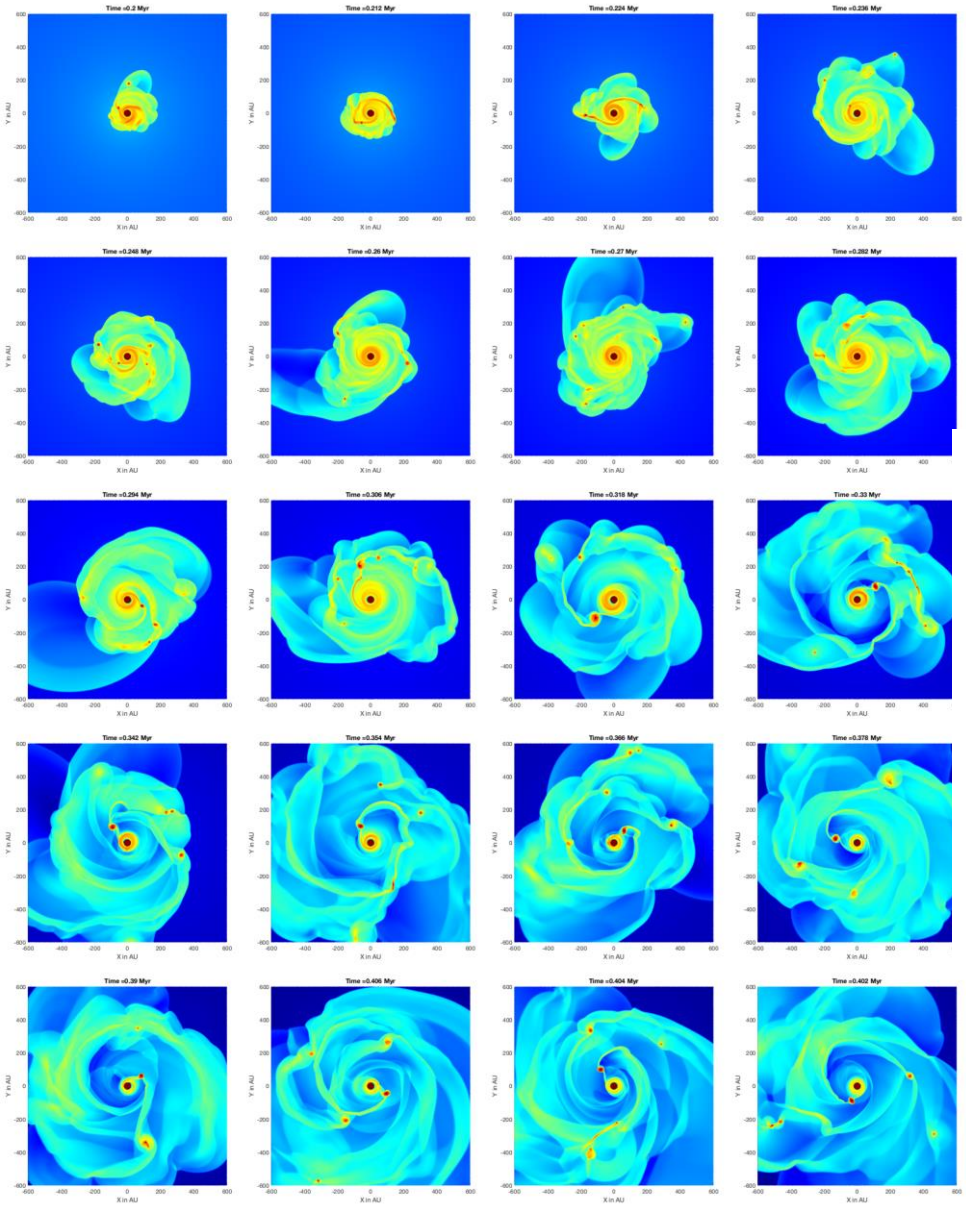
$$\frac{\partial}{\partial t}(\Sigma \boldsymbol{v}_p) + [\nabla \cdot (\Sigma \boldsymbol{v}_p \otimes \boldsymbol{v}_p)]_p = -\nabla_p \mathcal{P} + \Sigma \boldsymbol{g}_p + (\nabla \cdot \Pi)_p$$

$$\frac{\partial e}{\partial t} + \nabla_p \cdot (e \boldsymbol{v}_p) = -\mathcal{P}(\nabla_p \cdot \boldsymbol{v}_p) - \Lambda + \Gamma + (\nabla \boldsymbol{v})_{pp'} : \Pi_{pp'}$$

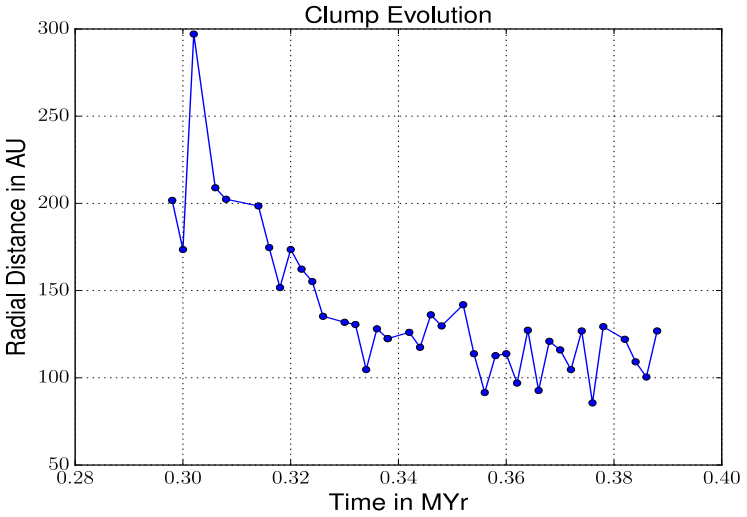
$$\Pi = 2\Sigma\nu \left(\nabla \boldsymbol{v} - \frac{1}{3}(\nabla \cdot \boldsymbol{v})\mathbf{e} \right)$$

Stress tensor



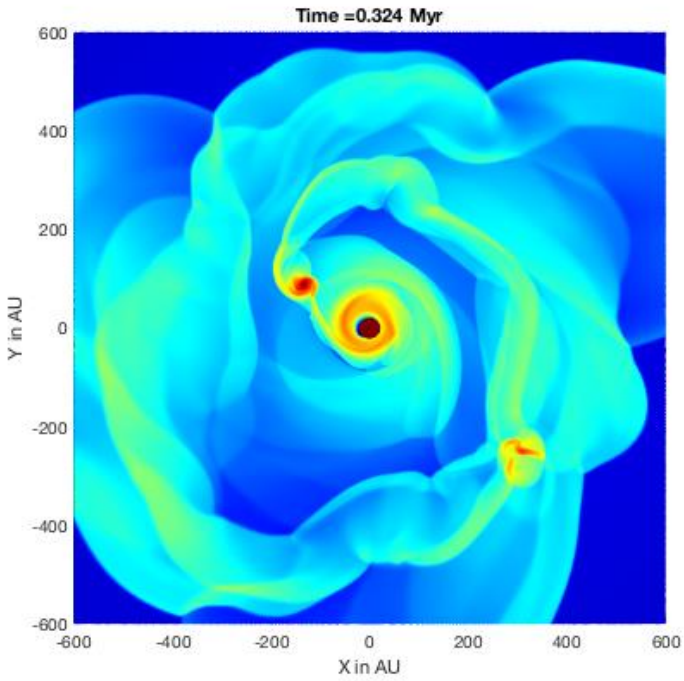


Migration of a massive clump



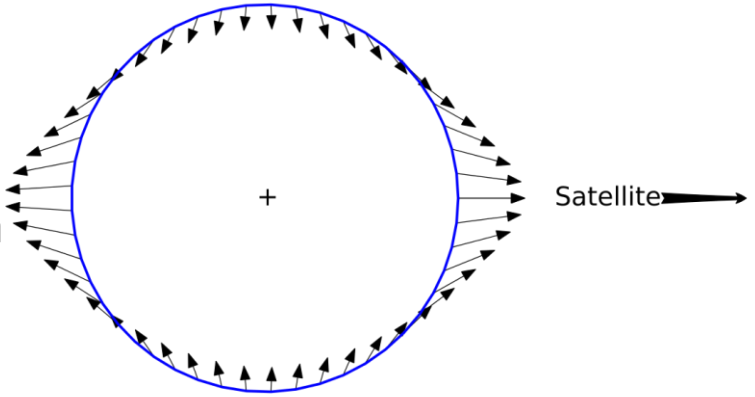
Migrating Embryo Model (Overview)

- Due to angular momentum transport by viscous forces make the clumps formed in wide orbits to come inner orbits.
- These clumps will then accrete more gas and dust as it move inward.
- The dust will get sediment in the core of clump, forming a solid core
- If the gas in the disk clears gap, the clump will go in to a stable orbit.
- Depending upon the tidal forces planet may or may not lose gas forming terrestrial and gas giants respectively



Tidal Forces and Hill Radius

- Tidal Forces arise due to the finite size of two objects gravitationally interacting.
- This exerts strain on each object. If strain is large enough it can rip apart a part of one object by other.
- Hence in the case of a clump formed in disk if clump is close to the star it will lose the mass to the star.

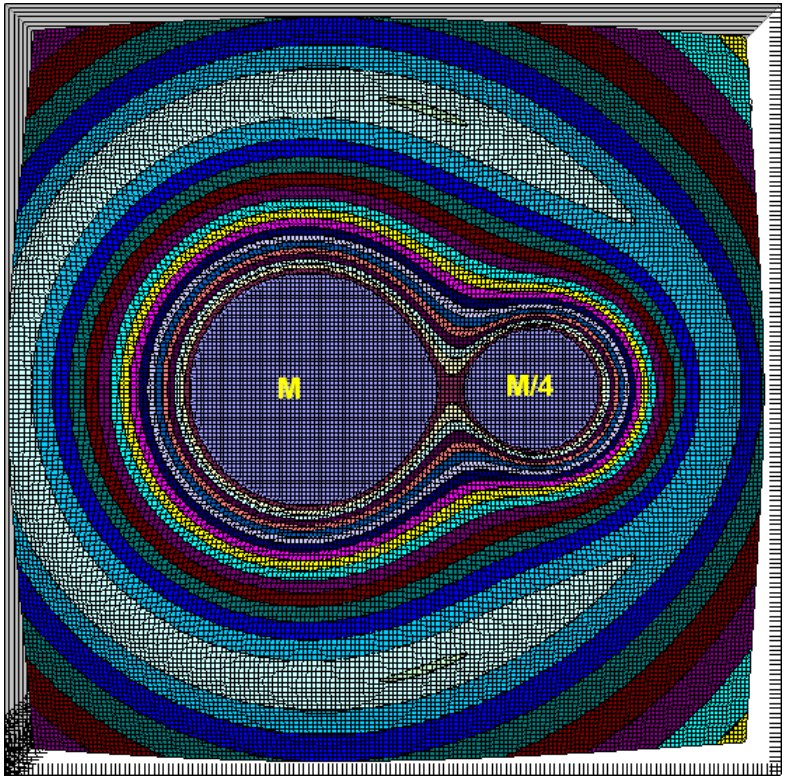


The radius at which gravitational acceleration due to one object become less than the other is called a Hill radius! (Radius of Influence)

Tidal Forces and Hill Radius

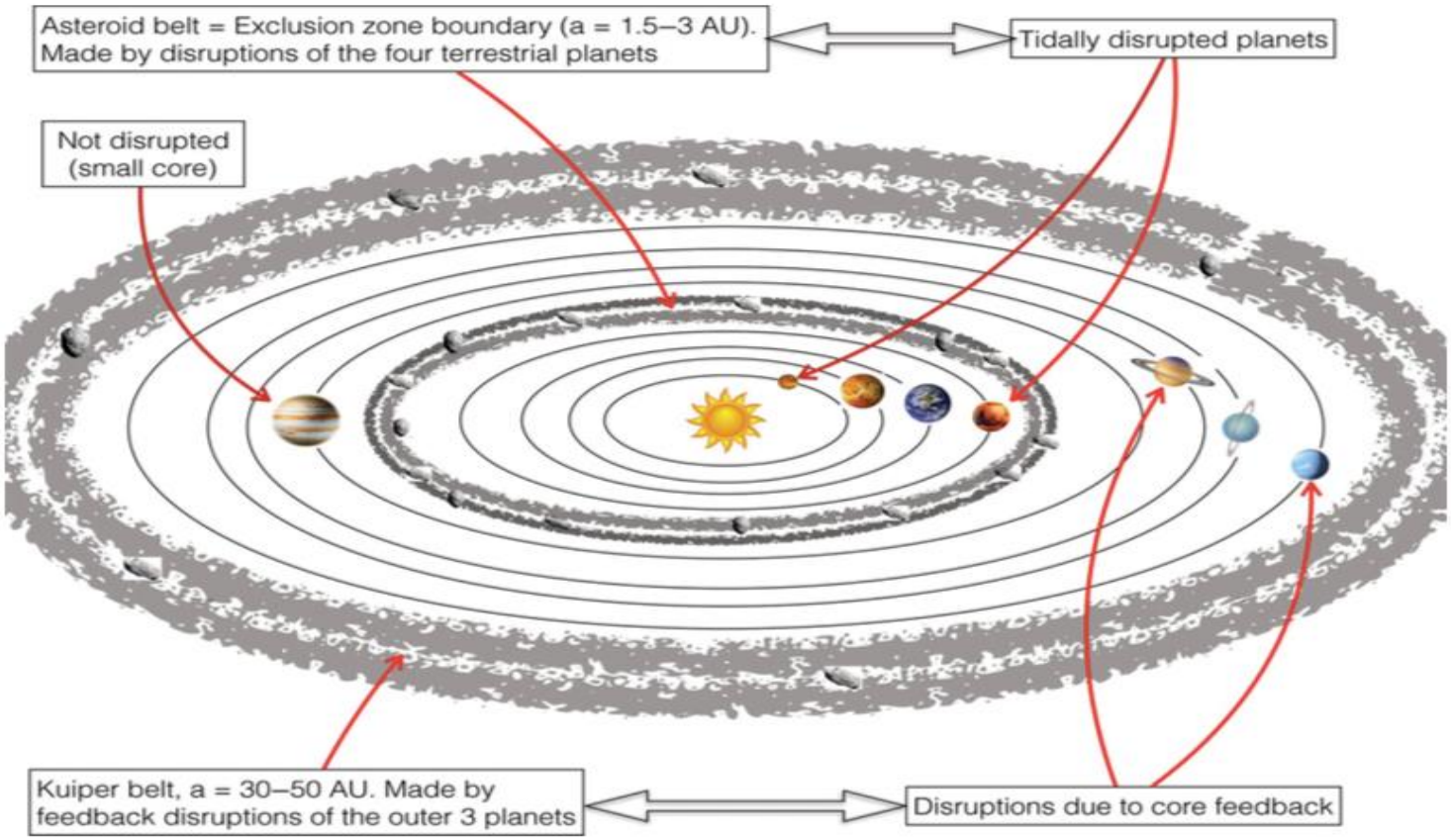
$$r \approx a\sqrt[3]{\frac{m}{3M}}$$

Hence if the radius of the clumps formed are **more than** Hill radius the mass between clump radius and Hill radius will get accreted by star



Pic Credit: Google

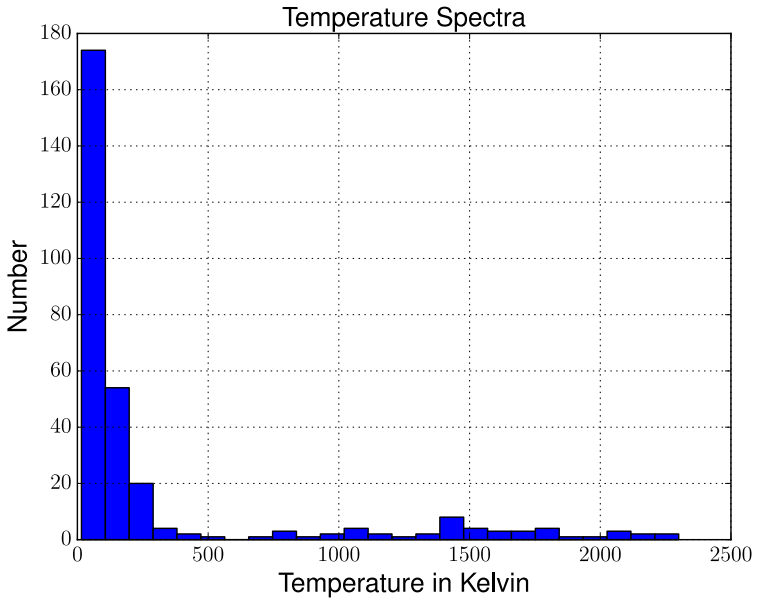
Tidally Downsized Migrating Embryos

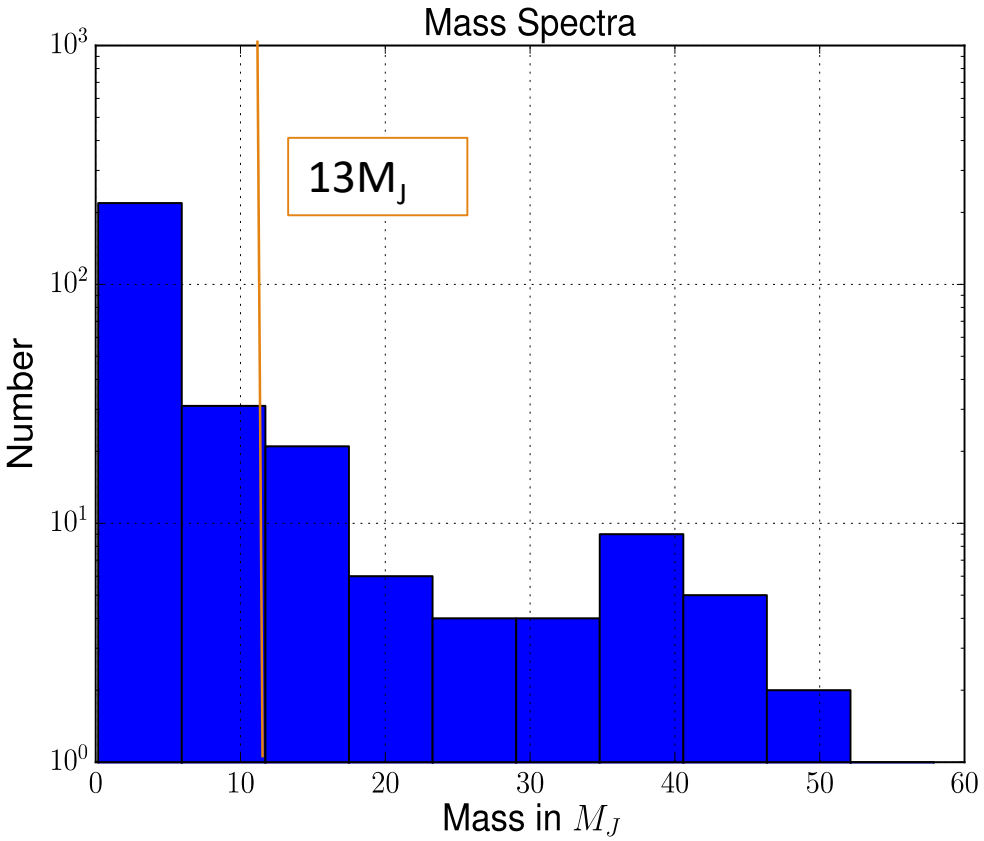


Nayakshin 2016

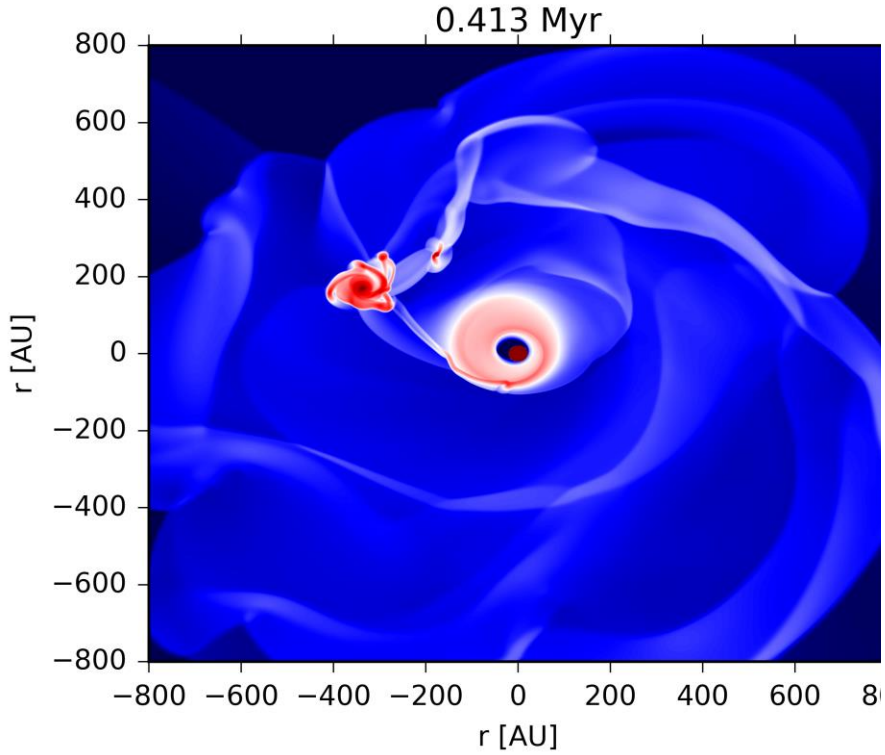
Migrating Embryo: A Versatile Theory

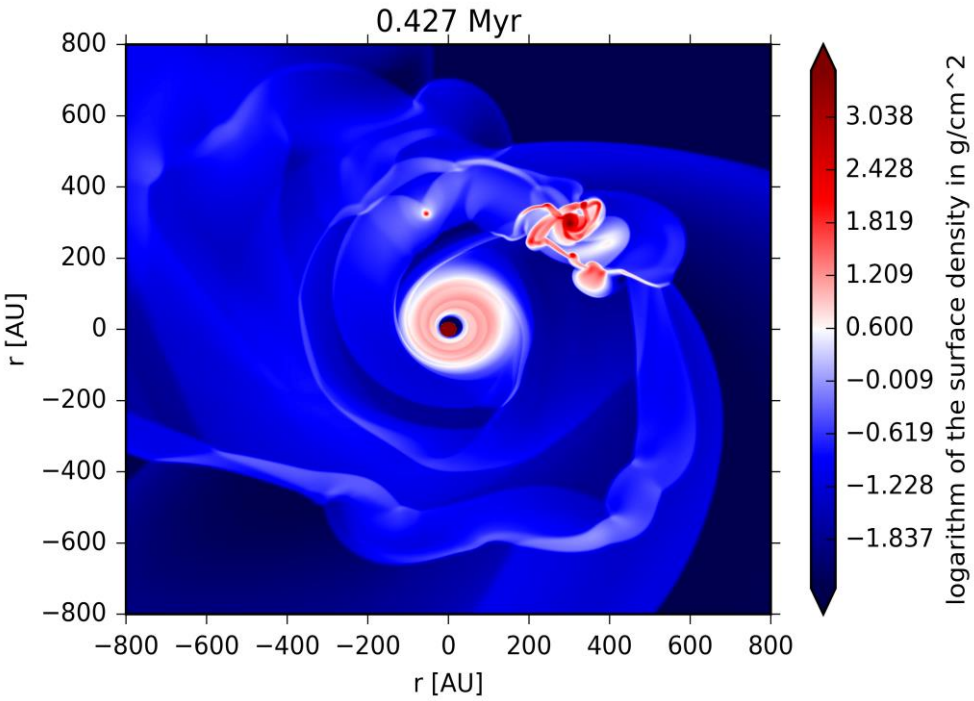
- It can not only explain the formation of planets, but the formation of Brown Dwarfs and Companion stars !!





Formation of a companion star in the disk





Conclusion

- A combination of Core accretion model and Gravitational instability model are required to explain the complete formation of Planets
- GI will create massive wide orbit planets and they will migrate inwards
- During this journey they will accrete dust to form a solid core
- The tidal forces will rip apart the gases to form terrestrial planets
- Those which survive tidal forces will form giant planets
- Migrating Embryo model can also explain the formation of brown dwarf type objects in the disks


"Consider again that dot. That's here. That's home. That's us.
On it everyone you love, everyone you know, everyone you ever heard of,
every human being who ever was, lived out their lives.
The aggregate of our joy and suffering, thousands of confident religions, ideologies,
and economic doctrines, every hunter and forager, every hero and coward,
every creator and destroyer of civilization, every king and peasant,
every young couple in love, every mother and father, hopeful child,
inventor and explorer, every teacher of morals, every corrupt politician,
every "superstar", every "supreme leader", every saint and sinner
in the history of our species lived there - on a mote of dust suspended in a sunbeam."
- Carl Sagan

THANK YOU