The Stellar Initial Mass Function (IMF)

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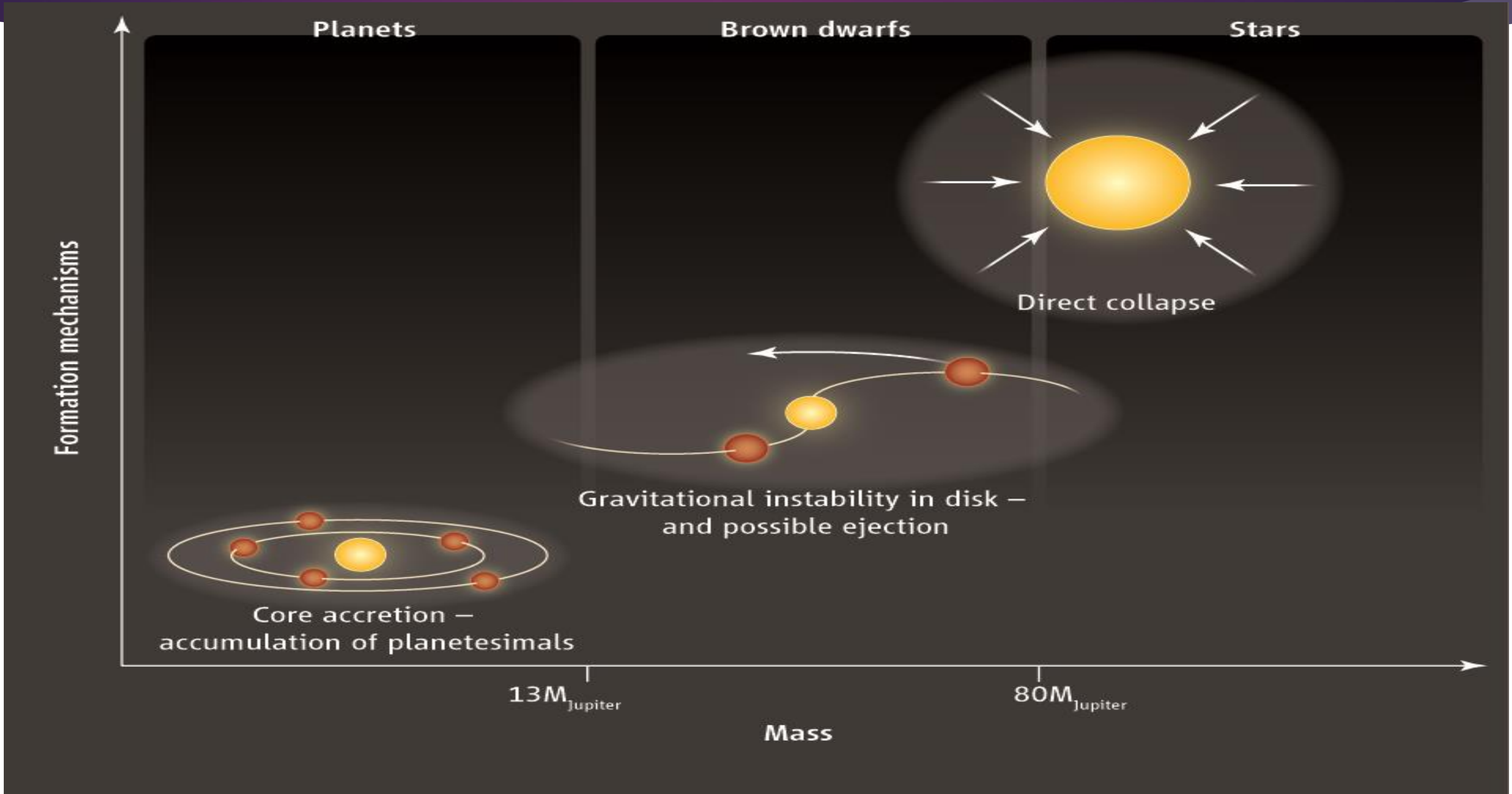
What is the IMF?

- The distribution of stellar and substellar object masses at the time they start their life
- A probability density function for masses, *f(m)*
- BUT…
- We only observe radiant flux directly
- Field stars have a very wide range of ages (~ 10 Gyr)
- Observing an individual star cluster reduces the age spread considerably (to ~ Myr) but sample size smaller

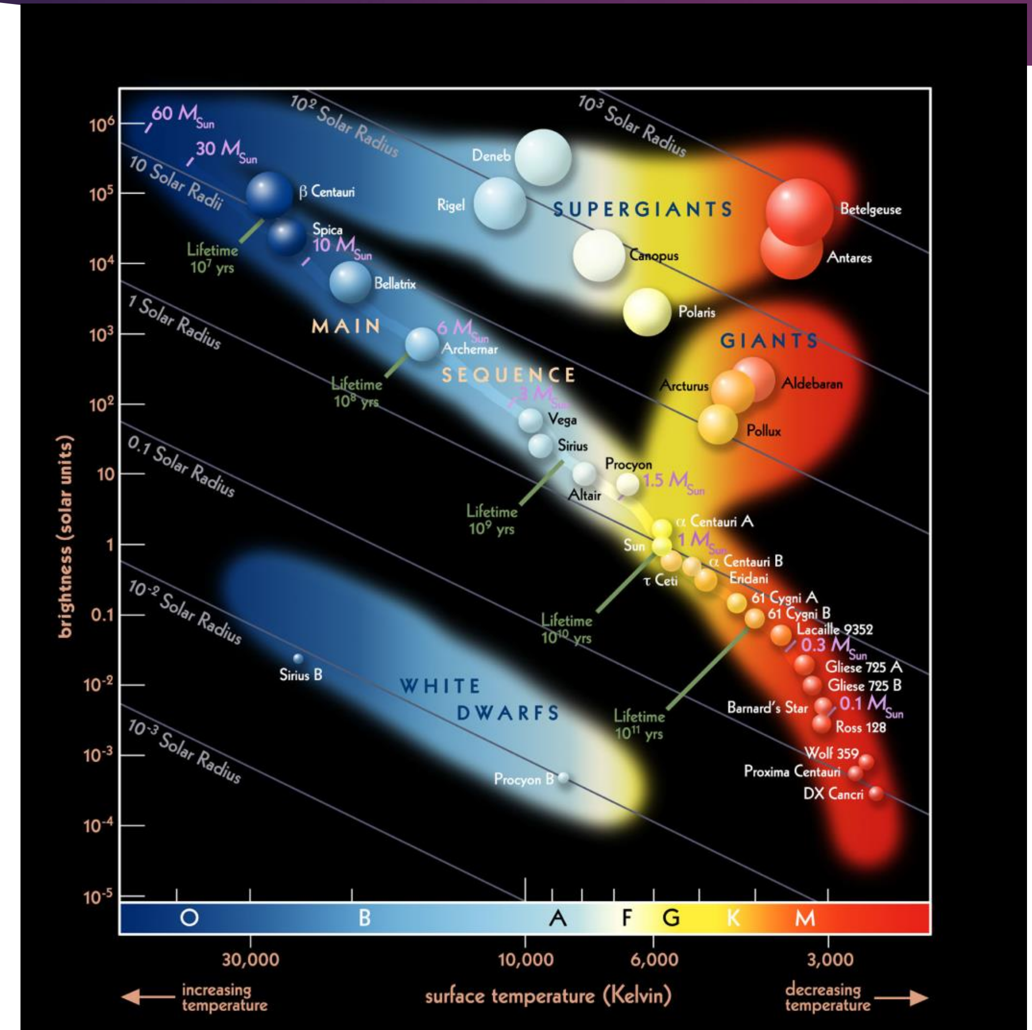
Mass Limits

- Maximum mass ~ 100 M_sun. Radiation pressure overcomes gravitational pressure
- Minimum stellar mass ~ 0.075 M_sun = 75 M_Jup. Enough gravitational pressure to sustain steady-state nuclear fusion
- Minimum substellar (brown dwarf) mass ~ 0.013 M_sun = 13 M_Jup. Enough gravitational pressure for some nuclear fusion
- Planets, asteroids have all masses < 13 M_Jup

Formation Mechanisms



Hertzsprung-Russell (HR) Diagram



Main sequence is the phase of steady-state nuclear fusion, when internal energy generation balances energy loss at the surface.

Position on main-sequence (Luminosity, Temperature) depends on mass *m*.

Young Stellar Clusters

- All stars and brown dwarfs formed at approximately the same time
- Even brown dwarfs may be detected since they are brightest when young
- A great laboratory for IMF studies if the sample size is large
- Need to worry about contamination, age uncertainty, possible age spread

IR view of the Orion Nebula Cluster – courtesy European Southern Observatory

New deep image of ONC

IR view of Orion Nebula Cluster.
Courtesy: ESO

New deep, wide near near-IR VLT HAWK-1 map, Drass et al. (2016)

~ 920 low mass stars
~ 760 brown dwarfs
~ 160 planemos

A multitude of very low mass objects from ejection from multiple systems during the early star-formation process?

New respect for substellar objects?

"Face it—in this town, either you're a star or you're just another brown dwarf."

Fig. 1 Cartoon from Mick Stevens published in the New Yorker magazine issue 01/08/1996 (Reprinted with permission by The Cartoon Bank)

Field Star IMF

- Search solar neighborhood stars where sample is considered complete (i.e., can see faintest objects)
- Convert luminosities to mass
- Need significant corrections for stellar evolution
- Need to assume a birthrate of stars *B(t)* – usually taken to be constant

Main-sequence lifetimes

Mass-luminosity relation is approximately $L \propto M^{3.3}$

Lifetime is then ~ available energy/energy loss rate $\tau \propto \frac{Mc^2}{L} \propto M^{-2.3}$

$$\tau = 10\left(\frac{M}{M_{sun}}\right)^{-2.3} \text{ Gyr} \qquad \begin{aligned} &= 13 \text{ Gyr for } 0.9\, M_{sun} \\ &= \;\; 5 \text{ Myr for } 10\, M_{sun} \end{aligned}$$

From PDMF to IMF for field stars

Consider stars of mass *m* and MS lifetime $\tau(m)$ forming at uniform rate over a time period *T*.

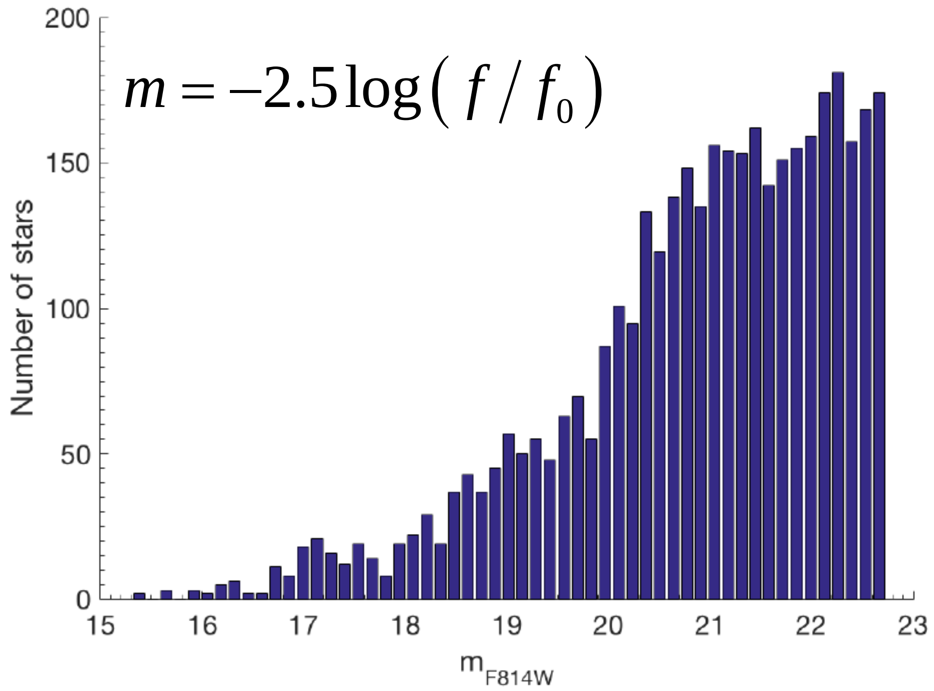
What is the correction factor to convert ΔN_{pdmf} to ΔN_{imf} if $M > 0.9\ M_{sun}$?

$$\Delta N_{\mathrm{imf}} = \Delta N_{\mathrm{pdmf}} \frac{T}{\tau(m)}$$

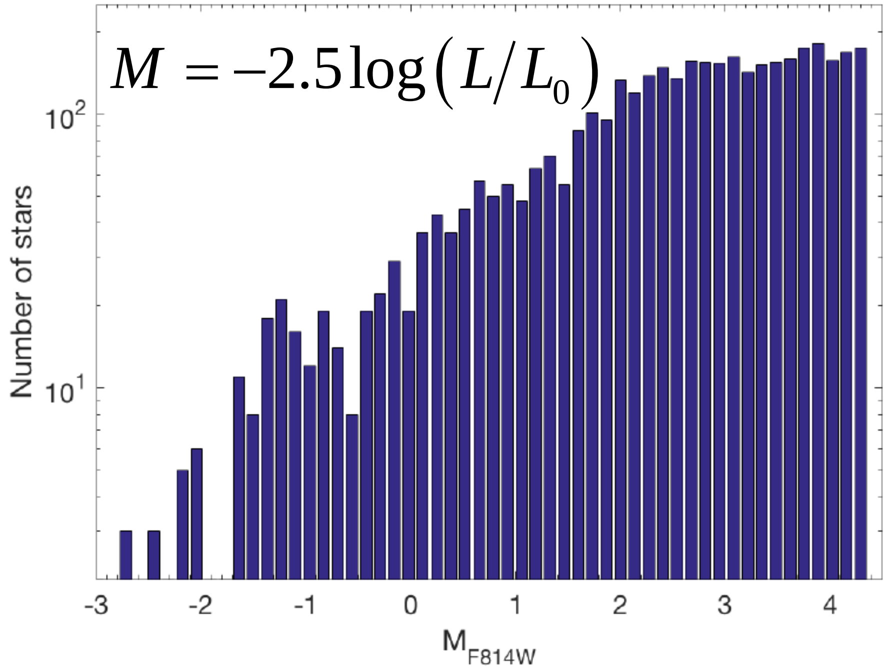
From Luminosity to Mass

Example, stellar cluster NGC 1711, Madaan et al. (2017)

flux **luminosity**



$$f = \frac{L}{4\pi d^2}$$

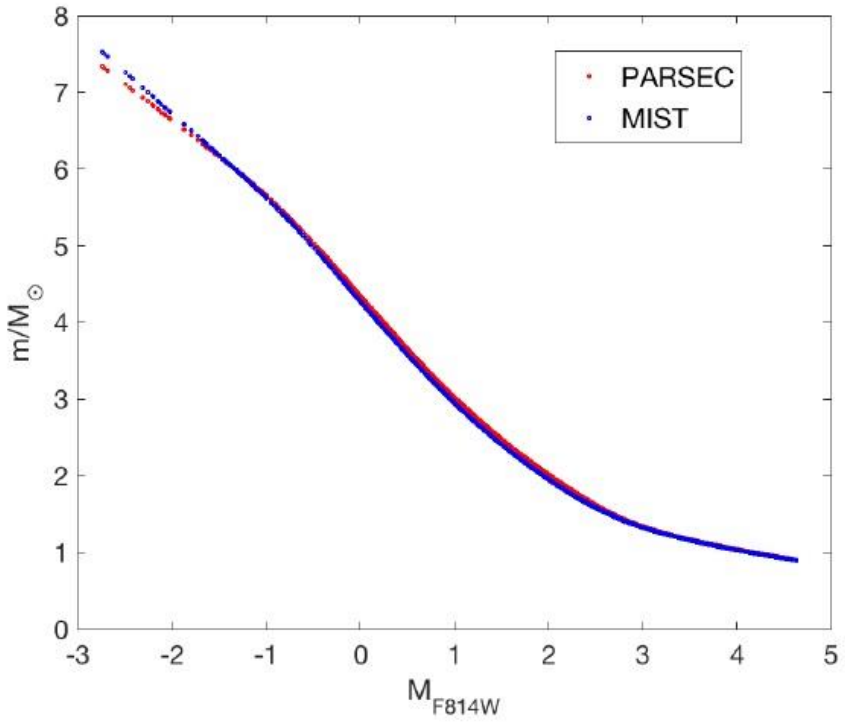


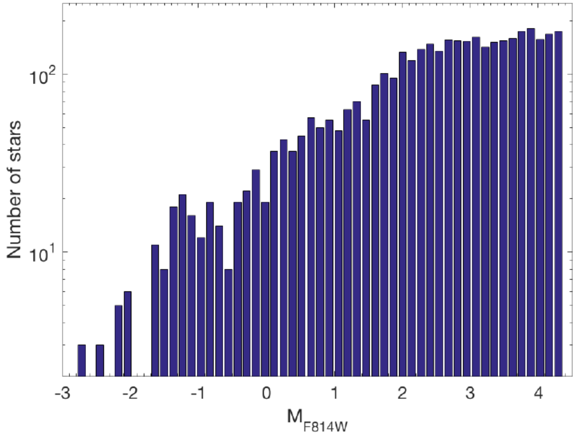
Apparent magnitude *m* relatable to flux *f*, units energy/time/area

Absolute magnitude *M* relatable to luminosity *L*, units energy/time

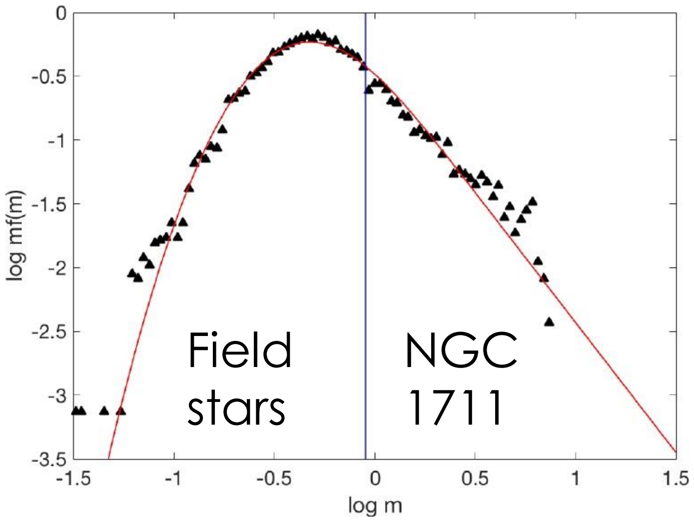
From Luminosity to Mass

Use an empirically calibrated theoretical model to convert luminosity to mass



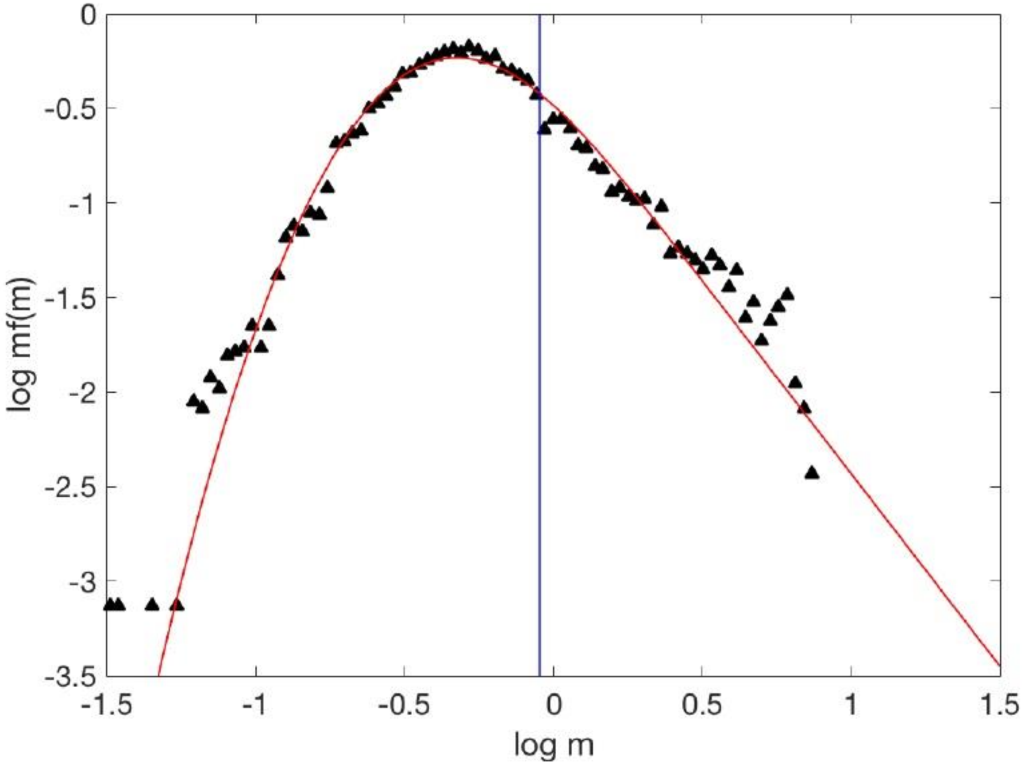


NGC 1711



Madaan et al. (2017)

Probability Distribution Functions



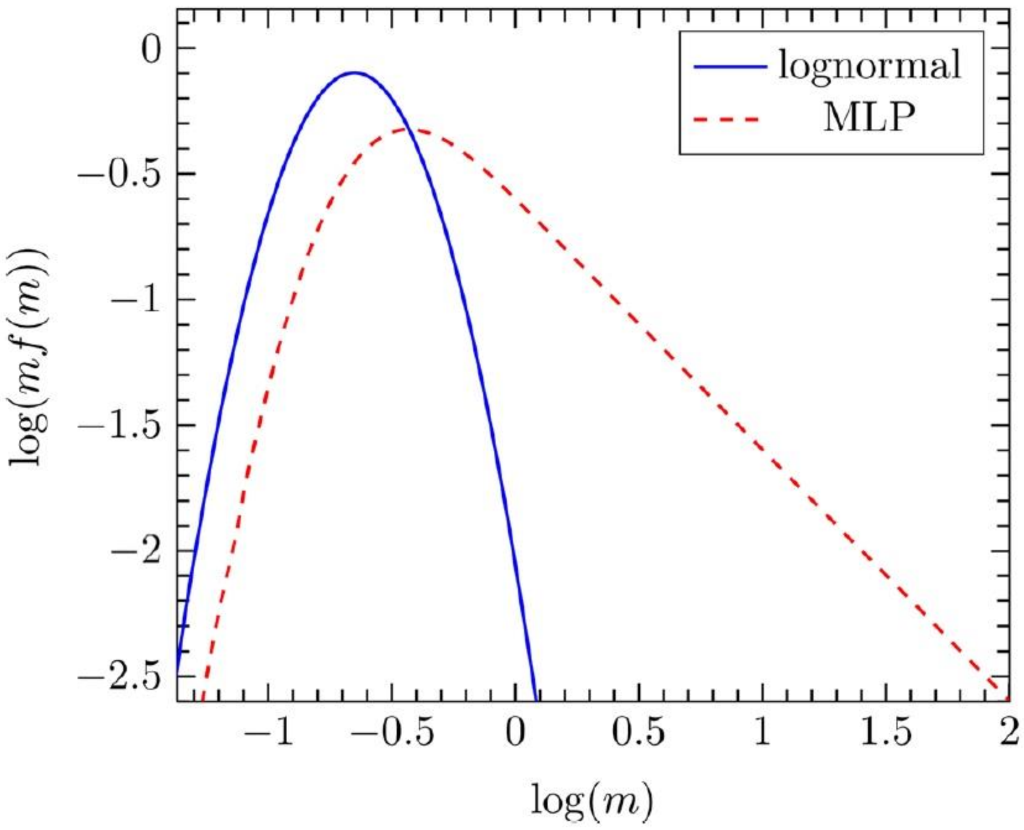
$$f(m) \equiv \frac{dN'}{dm} \text{ where } dN' = dN/N_{tot}$$

$$m\, f(m) = m\frac{dN'}{dm} = \frac{dN'}{d\ln m}$$

Plotting $\dfrac{\Delta N'}{\Delta \log m}$ vs. $\Delta \log m$ is the norm.

$$(\Delta \log m = \Delta \ln m / \ln 10)$$

Probability Distribution Functions



Possible forms of *f(m)*:

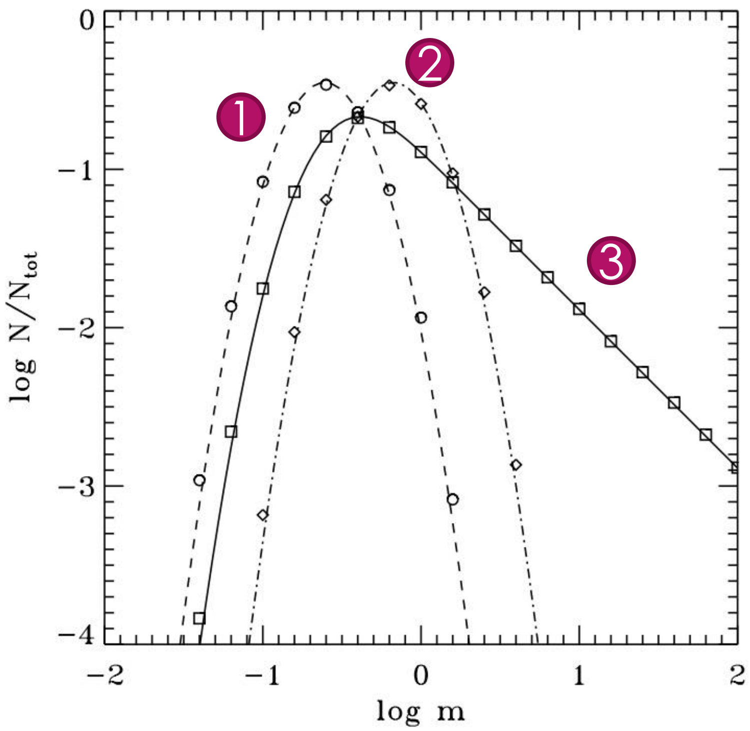
Lognormal: $$f(m) = \frac{1}{m\sqrt{2\pi}\sigma} \exp\left(-\frac{(\ln m - u)^2}{2\sigma^2} \right)$$

Modified Lognormal Power Law (MLP):

$$f(m) = \frac{\alpha}{2} \exp\left(\alpha\mu_0 + \alpha^2\sigma_0^2/2 \right) m^{-(1+\alpha)} \operatorname{erfc}\left(\frac{1}{\sqrt{2}} \left(\alpha\sigma_0 - \frac{\ln m - \mu_0}{\sigma_0} \right) \right)$$

Asymptotically $\propto m^{-(1+\alpha)}$

Modified Lognormal Power Law (MLP)



1. An initial lognormal
2. Lognormal plus exponential growth for fixed time
3. MLP: lognormal plus exponential growth for an exponential distribution of times

$$f(m) = \frac{1}{\sqrt{2\pi}\sigma m} \exp\left[-\frac{(\ln m - \mu)^2}{2\sigma^2} \right].$$

$$m = m_0 e^{\gamma t},$$

$$f(t) = \delta e^{-\delta t}$$

Modified Lognormal Power-Law (MLP) Distribution

$$f(m) = \frac{\alpha}{2} \exp\left[\alpha \mu_0 + \alpha^2 \sigma_0^2 / 2 \right] m^{-(1+\alpha)}$$

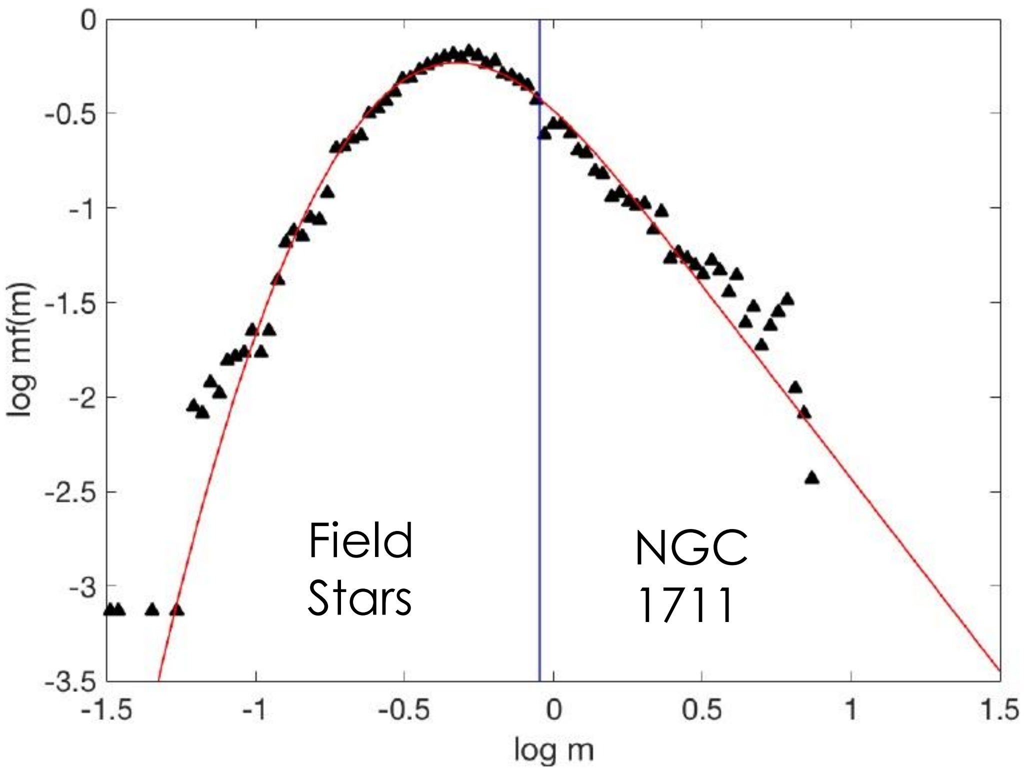
$$\times \operatorname{erfc}\left[\frac{1}{\sqrt{2}} \left(\alpha\sigma_0 - \frac{\ln m - \mu_0}{\sigma_0} \right) \right],$$

where $\alpha = \delta/\gamma$. 3 parameters: μ_0, σ_0, α.

Power-law index $\alpha=\delta/\gamma$ is the ratio of characteristic growth time of stars to the characteristic time of accretion termination.

Basu et al. (2015)

Modified Lognormal Power Law (MLP)



Madaan et al. (2017)

$$f(m) = \frac{\alpha}{2} \exp\left(\alpha\mu_0 + \alpha^2\sigma_0^2/2\right) m^{-(1+\alpha)} \operatorname{erfc}\left(\frac{1}{\sqrt{2}}\left(\alpha\sigma_0 - \frac{\ln m - \mu_0}{\sigma_0}\right)\right)$$

Best fit parameters for this data set are

$$\mu_0 = -1.10, \sigma_0 = 0.55, \alpha = 2.04$$

- Most stars have masses not much above the limit for sustained nuclear fusion
- Self-regulation of mass accumulation process?
- For high mass end, is there a physical interpretation of slope α?

Thank You!

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